1. Homework 1

Due: In Lecture 9-21

Problem 1. Show that if $A \subset \mathbb{R}^n$ is a rectangle and $f : A \to \mathbb{R}$ is continuous, then f is Riemann integrable on A.

Problem 2. Show that if f and g are Riemann integrable on $A \subset \mathbb{R}^n$ a rectangle, then so is $f + g$, and

$$\int_A (f+g) = \int_A f + \int_A g,$$

Problem 3. Let $f(x,y) = x^2y^2$. Show that the set of critical points of f consists of the union of the x and y axes, and that they are all degenerate. Sketch the graph of f to see how f behaves.

Problem 4. Show that the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x,y) = (e^x + e^y, e^x + e^{-y})$$

is locally invertible about any point $(a,b)\epsilon\mathbb{R}^2$, and compute the Jacobian matrix of the inverse map.

Problem 5. Same for the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x,y) = (e^x + e^y, e^x - e^y).$$

In this case, the whole map f is invertible, with an easily computed inverse, g. Compute g and its Jacobian matrix, and check that the Jacobians of f and g really are inverses of each other.

Problem 6. Show that the system of equations

$$3x + yz + u^2 = 0$$

$$xy + 2z + u = 0$$

$$2x + 2y - 3z + 2u = 0$$

can be solved for x, y, u in terms of z ;

for x, z, u in terms of y;

for x, y, z in terms of u;

but not for y, z, u in terms of x.

Problem 7. Define $f : \mathbb{R}^2 \times \mathbb{R}^1 \to \mathbb{R}^1$ by

$$f(x,y,z) = z^2x + e^z + y,$$

and note that $f(1,-1,0) = 0$.

Use the Implicit Function Theorem to conclude that there is a function $g : \mathbb{R}^2 \to \mathbb{R}^1$ defined in some neighborhood of $(1,-1)$ such that $g(1,-1) = 0$ and such that $f(x,y,g(x,y)) = 0$ for all (x,y) in that neighborhood.

Then calculate $\frac{\partial g}{\partial x}(1,-1)$ and $\frac{\partial g}{\partial y}(1,-1)$.

Problem 8. Let $F(x,y)$ be of class C^2, and suppose that $F(x,f(x)) = 0$ and $\frac{\partial F}{\partial y}(x,f(x)) \neq 0$ for all $x\epsilon\mathbb{R}$. Calculate f' and f" in terms of F and its derivatives.